Math 213, Fall 2025, Exam 3
Read Carefully, Write Neatly, Sketch Accurately

1. (12 points) Sketch the region and convert to an equivalent polar integral

$$\int_{\sqrt{2}}^{2}\int_{\sqrt{4-y^2}}^{y} dxdy.$$ DO NOT EVALUATE

2. (12 points) Evaluate by changing the order of integration appropriately (hint: reverse x and y)

$$\int_0^9\int_0^1\int_{2y}^{2}\frac{2\cos(x^2)}{\sqrt{z}}dxdydz$$

3. (10 points) Set up the triple integral in cylindrical coordinates to find the volume of the region bounded below by triangle in the x-y plane with sides $x=\sqrt{3}\,y$; $x=\sqrt{3}$ and $y=0$ and on top by the surface $z=6-x^2$. DO NOT EVALUATE.

4. (8 points) Set up the triple integral in spherical coordinates for the volume of the region above the inverted cone (i.e. vertex at the origin) that makes an angle of $\frac{\pi}{6}$ with the z-axis and inside the sphere of radius 4. DO NOT EVALUATE.

5. (12 points) Evaluate the integral $\iint_R\left(\sqrt{\frac{y}{x}}+\sqrt{xy}\right)dxdy$ where R is the region bounded by $xy=1$; $xy=9$; $y=x$; and $y=4x$ by using the transformation $x=\frac{u}{v}$; $y=uv$ to rewrite the integral in terms of u and v.

6. (8 points) Evaluate $\int(2x-y+z)\,ds$ along the straight line from (2,1,0) to (2,0,1)

7. (8 points) Find the (counterclockwise) flux of the field $\vec{\mathbf{F}}=x\vec{\mathbf{i}}+2x\vec{\mathbf{j}}$ directly, without using Green's Theorem) across the circle of radius 2 around the origin.

8. (8 points) Show that $\vec{\mathbf{F}}=e^x\sin y\,\vec{\mathbf{i}}+(e^x\cos y+2y)\,\vec{\mathbf{j}}$ is a gradient field (i.e., a conservative field) and find it potential function.

9. (10 points) Use Green's Theorem to find the counterclockwise circulation for the field $\vec{\mathbf{F}}=x\vec{\mathbf{i}}+2x\vec{\mathbf{j}}$ on the unit circle around the origin.

10. (12 points) Find the area of the band cut from the paraboloid $x^2+y^2-z=0$ by the planes $z=4$ and $z=9$.

$$x=r\cos\theta;\ y=r\sin\theta; \qquad x=\rho\cos\theta\sin\phi;y=\rho\sin\theta\sin\phi;z=\rho\cos\phi$$

$$\iint_R f(x,y)\,dA=\iint_R f(x,y)\,dxdy=\iint_R f(r,\theta)\,rdrd\theta$$

$$\iiint_D f(x,y,z)\,dxdydz=\iiint_D f(r,\theta,z)\,r\,drd\theta dz=$$

$$\iiint_D f(\rho,\phi,\theta)\ \rho^2\sin\phi\ d\phi d\theta d\rho=$$

$$\iiint_D f(u,v,w)\,J(u,v,w)\,dudvdw \text{ where } J(u,v,w)=\begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix}$$

$$\int_C f(x,y,z)\,ds=\int_C f(x,y,z)\left|\frac{dr}{dt}\right|dt=\int_C f(x,y,z)\,|v(t)|\,dt$$

$$\int_C \vec{\mathbf{F}}\cdot\vec{\mathbf{T}}ds=\int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}=\int_C \vec{\mathbf{F}}\cdot\frac{d\vec{\mathbf{r}}}{dt}dt=\int_C Mdx+Ndy+Pdz=\int_C\left(M\frac{dx}{dt}+N\frac{dy}{dt}+P\frac{dz}{dt}\right)dt$$

$$\int_C \vec{\mathbf{F}}\cdot\vec{\mathbf{n}}ds=\int_C Mdy-Ndx=\int_C\left(M\frac{dy}{dt}-N\frac{dx}{dt}\right)dt$$

$$\oint \vec{\mathbf{F}}\cdot\vec{\mathbf{T}}ds=\iint\left(N_x-M_y\right)dxdy=\iint\left(\nabla\times\vec{\mathbf{F}}\right)\cdot\vec{\mathbf{k}}dxdy$$

$$\oint \vec{\mathbf{F}}\cdot\vec{\mathbf{n}}ds=\iint\left(M_x+N_y\right)dxdy=\iint\left(\nabla\cdot\vec{\mathbf{F}}\right)dxdy$$

$$\vec{\mathbf{r}}(u,v)=f(u,v)\,\vec{\mathbf{i}}+g(u,v)\,\vec{\mathbf{j}}+h(u,v)\,\vec{\mathbf{k}}$$

$$\iint d\sigma\colon \iint\left|\vec{\mathbf{r}}_u\times\vec{\mathbf{r}}_v\right|dudv;\quad \iint\frac{|\nabla F|}{|\nabla F\cdot\vec{\mathbf{k}}|}dxdy;\quad \iint\sqrt{1+f_x^2+f_y^2}\,dxdy$$

$$\oint \vec{\mathbf{F}}\cdot\vec{\mathbf{n}}d\sigma=\iint \vec{\mathbf{F}}\cdot\left(\vec{\mathbf{r}}_u\times\vec{\mathbf{r}}_v\right)dudv$$

$$\oint \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}=\iint\left(\nabla\times\vec{\mathbf{F}}\right)\cdot\vec{\mathbf{n}}d\sigma;\qquad \iint\vec{\mathbf{F}}\cdot\vec{\mathbf{n}}d\sigma=\int\iint\left(\nabla\cdot\vec{\mathbf{F}}\right)dV$$